MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

DIM5058 – MATHEMATICAL TECHNIQUES 1

(For DIT students only)

27 OCTOBER 2017
09.00 am - 11.00 am
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **3** pages with **4** questions. Key formulae are given in the Appendix.
2. Answer **ALL** questions.
3. Write your answers in the answer booklet provided.
4. All necessary working steps must be shown.

Question 1

a) Rationalize the denominator of $\dfrac{\sqrt{5}+5}{1-\sqrt{5}}$ and simplify your answer. (4 marks)

b) Solve the equation $3xy^2 - 75x - 2y^2 = -50$. (5 marks)

c) Solve the equation $5x^2 + 2x - 7 = 0$ by using **completing the square method**. (6 marks)

d) Solve the absolute inequality $|8x-5| + 2x - 1 < 10 + 2x$ and represent the answer on an interval notation. (5 marks)

[TOTAL 20 MARKS]

Question 2

a) Given that $g(x) = \dfrac{3x-5}{4}$ and $h(x) = \dfrac{4+3x}{9}$.

i) Find $(g+h)(x)$. (4 marks)

ii) Evaluate $g^{-1}(1)$. (3 marks)

b) Given a quadratic function $f(x) = (x+2)(1-x)$.

i) Determine whether the parabola opens upward or downward. (2 marks)

ii) Find the vertex of the parabola. (3 marks)

iii) Find the x – intercept(s). (3 marks)

iv) Find the y – intercept. (1 marks)

v) Sketch the parabola and label the necessary points. (4 marks)

[TOTAL 20 MARKS]

Continued...

Question 3

a) Given that $A=\begin{bmatrix}3 & -7 & 4\\ 4 & 5 & 8\\ 8 & -6 & 10\end{bmatrix}$ and $B=\begin{bmatrix}1 & 10 & -7\\ 3 & 1 & -9\\ -2 & 11 & 1\end{bmatrix}$.

i) Find $3A+4B$. (4 marks)

ii) Find B^T-A. (3 marks)

b) Given that $A=\begin{bmatrix}5 & 4\\ 4 & 3\end{bmatrix}$ and $M=\begin{bmatrix}-14 & 20\\ -12 & 17\end{bmatrix}$.

i) If $AA^{-1}=A^{-1}A=I$, find A^{-1}. (3 marks)

ii) From part (i), show that $A^{-1}M^T=\begin{bmatrix}122 & 104\\ -156 & -133\end{bmatrix}$. (5 marks)

c) A furniture company produces three types of desks: a traditional model, modern model and deluxe model. Each desk is manufactured in three stages: cutting, construction and finishing. The time requirements for each model and manufacturing stage are given in the following table.

Manufactured Process	Model		
	Traditional	Modern	Deluxe
Cutting	2	3	2
Construction	2	1	3
Finishing	1	1	2

Each week the company has available a maximum of 100 hours for cutting, 100 hours for construction and 65 hours for finishing. [Hint: Let x, y and z be the number of desks for traditional, modern and deluxe model respectively.]

i) Represent the above information in the form of $AX=B$. (2 marks)

ii) From part (i), solve the value of x, y and z by using **Cramer's Rule**. (13 marks)

[TOTAL 30 MARKS]

Continued...

Question 4

a) Write the first five terms of the sequence whose general form is $a_n = \dfrac{3a_{n-1}+1}{(n+1)!}$ and given the first term is $a_1 = -5$. (4 marks)

b) Find the sum for $\sum_{x=1}^{6} \dfrac{(2x+5)^2}{2^x}$. (4 marks)

c) Insert three arithmetic mean between number 13 and 161. (3 marks)

d) At the corner section of a football stadium, there are 8 seats in the first row and 35 rows in total. Each successive row contains 4 additional seats.

  i) Find the first term and the common difference. (2 marks)

  ii) Calculate the number of seats in the last row. (2 marks)

  iii) Find the total number of seats in the corner section. (2 marks)

e) Sarah has been hired by PQR company with an annual salary of RM28,000 and is expected to receive an annual increase of 6%. What will Sarah's annual salary be in her ninth year of service? (3 marks)

f) The sum to infinity of a geometric series is 280. If the first term is 182, find the common ratio. (4 marks)

g) Find the 4th and 9th term for the binomial expansion $(2x^3 + y^2)^{10}$. (6 marks)

[TOTAL 30 MARKS]

End of Page.

APPENDIX – KEY FORMULA

Inequalities:

Inequality	***Solution***
$\|x\| < a$	$-a < x < a$
$\|x\| \le a$	$-a \le x \le a$
$\|x\| > a$	$x < -a \quad or \quad x > a$
$\|x\| \ge a$	$x \le -a \quad or \quad x \ge a$

Completing the square: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Quadratic formula: If $\mathbf{ax^2 + bx + c = 0}$ where $a \neq 0$, then, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Standard form of a quadratic function: $f(x) = a(x-h)^2 + k,\ a \neq 0$

Determinant of a 2 × 2 matrix	***Determinant of a 3 × 3 matrix***
$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$	$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $= a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$
Inverse of a 2 × 2 matrix	***Inverse of a 3 × 3 matrix***
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.	$A^{-1} = \dfrac{1}{\|A\|}\left[c_{ij}\right]^T$ $A^{-1} = \dfrac{1}{\|A\|}\begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$ $A^{-1} = \dfrac{1}{\|A\|}adj\ A$ where $\left[c_{ij}\right]^T$ is called the adjoint of A (adj A). c_{ij} of the entry $a_{ij} = (-1)^{i+j}M_{ij}$

Cramer's Rule for 2 × 2 matrix	*Cramer's Rule for 3 × 3 matrix*
If $\begin{matrix} a_1x+b_1y=c_1 \\ a_2x+b_2y=c_2 \end{matrix}$ then $x=\dfrac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$ and $y=\dfrac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$, where $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$	If $\begin{matrix} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \\ a_3x+b_3y+c_3z=d_3 \end{matrix}$ then $x=\dfrac{D_x}{D}$, $y=\dfrac{D_y}{D}$, $z=\dfrac{D_z}{D}$ where $D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $D_x=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$, $D_y=\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$, $D_z=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

Arithmetic sequence	*Geometric sequence*
$a_n=a_1+(n-1)d$	$a_n=a_1r^{n-1}$, $S_n=\dfrac{a_1(1-r^n)}{1-r}$
$S_n=\dfrac{n}{2}(a_1+a_n)$	$S_\infty=\dfrac{a_1}{1-r}, \lvert r\rvert<1$

Binomial Theorem

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k;\quad n\geq 1$$

The $(r+1)^{st}$ term in the expansion of $(a+b)^n$ is $\dbinom{n}{r}a^{n-r}b^r$.